Simple Harmonic Motion

Purpose

Using Arduino accelerometer with "Lab in Your Pocket" app to investigate the simple harmonic motion of a pendulum.

Theory

- Simple harmonic motion (SHM) describes periodic motion, also known as oscillation. Note that SHM is no longer part of HKDSE syllabus.
- In SHM, there exists a **restoring** force which tends to pull the object towards the equilibrium of oscillation. The force is restoring because it acts opposite to the object's displacement, and is directly proportional to the displacement. The equation is written as

$$F_{net} = -kx$$

  where F_{net} is the restoring force, k is the constant of the oscillation, x is the displacement, the negative sign denotes that the direction of net force is opposite to that of the displacement.
- Solving differential equation gives the followings:

$$x(t) = A\cos(\omega t - \varphi)$$
$$v(t) = -A\omega\sin(\omega t - \varphi)$$
$$a(t) = -A\omega^2\cos(\omega t - \varphi)$$

  where A refers to the amplitude of oscillation.
- Assuming that the motion starts from the extreme point at time = 0, the phase difference φ is 0. It represents the following features of an SHM:
  - At extreme points ($x = A$), the object stops moving for one instant ($v = 0$) and the restoring force is maximum ($a = -A\omega^2$).
  - At equilibrium point ($x = 0$), the object moves fastest ($v = -A\omega$) and the restoring force is minimum ($a = 0$).
- In this experiment, a slightly swinging pendulum is used to resemble an SHM given the **oscillation angle is small** – this is to ensure negligible vertical motion.
- The following relation between the period of SHM and the length of string used to hang the pendulum will be studied:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

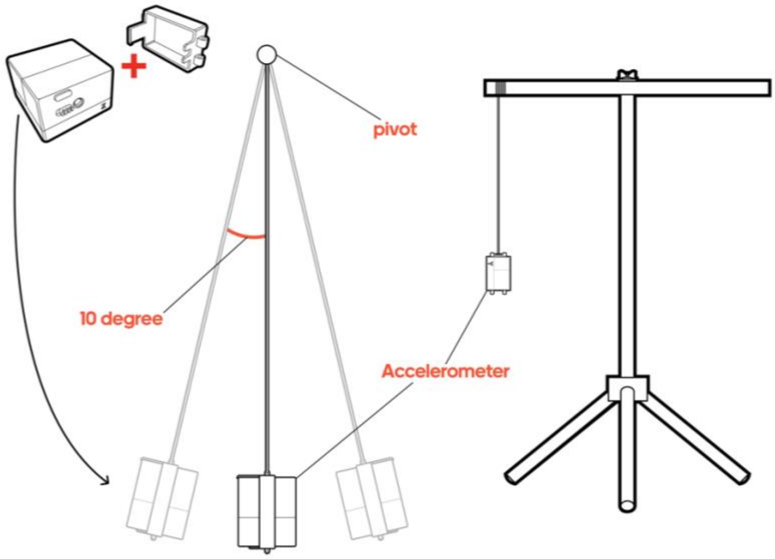
  where T is the period, l is the length of string, g is gravitational acceleration = $9.81ms^{-2}$.
- The Arduino accelerometer and Lab in Your Pocket app can display acceleration in 3 axes and the magnitude at a frequency of 20Hz.

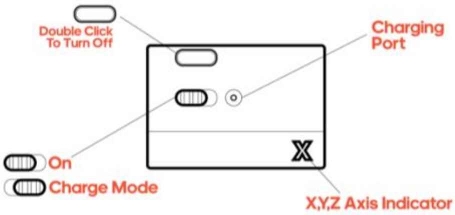
Apparatus

- A mobile device with "Lab in Your Pocket" app

- An Arduino accelerometer (provided by PolyU)
- A string

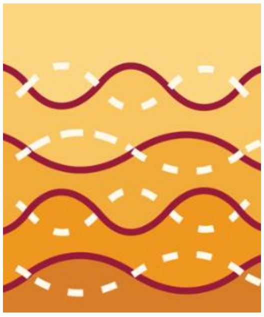
Setup



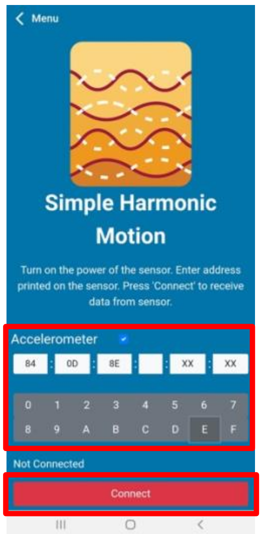


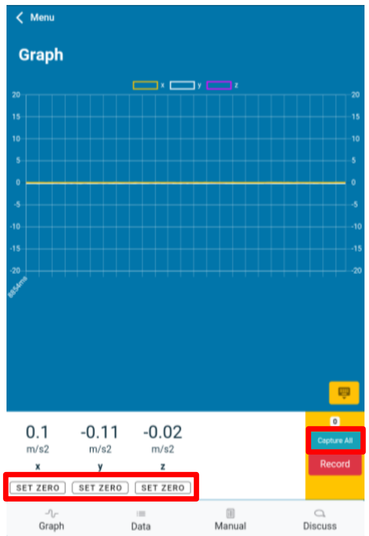
Procedure

<u>Set up the Experiment</u>



1. Place the accelerometer on a flat surface.
2. Turn on the Arduino accelerometer by pressing the power button once.
3. Open "Lab in Your Pocket" app in the mobile device and choose "Circular Motion".
4. Register the accelerometer with the address printed on it. Press "Connect" to connect the sensors. When the connection finishes, the app will automatically jump into a real-time graph of the acceleration in different axes.





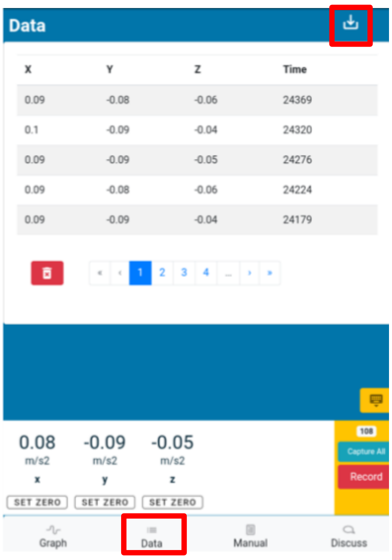
5. If necessary, press “Set Zero” button on the app interface to remove offsets.
6. Find and identify the axes and +/- directions of the accelerometer by moving it in all directions.
7. Tie the accelerometer with the string attached to ceiling. Measure the length l from the ceiling to the centre of the accelerometer.

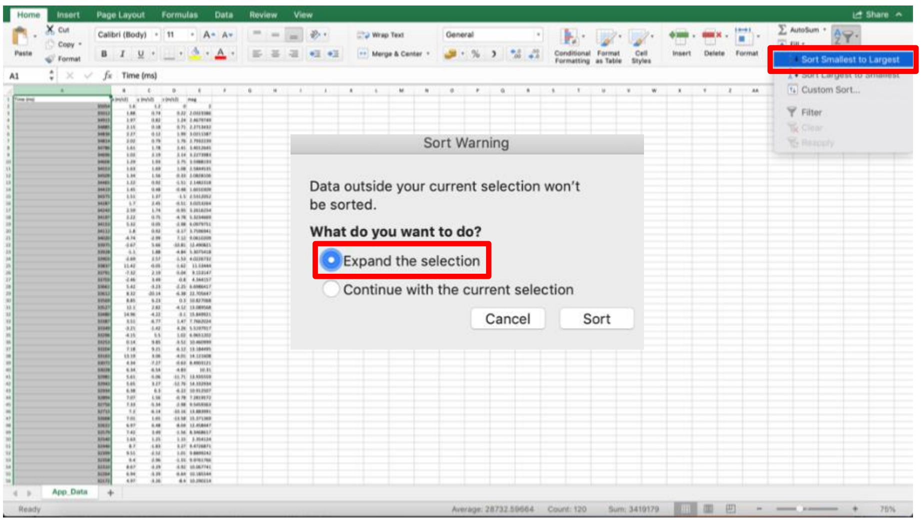
<u>Testing for SHM</u>

8. Start recording data by pressing “Capture All” in the app. The number above the button shows the data points collected.
9. Slightly pull the accelerometer sideway with no more than 10° from the normal. Release the accelerometer and allow it to swing for 10 times. Stop the measurement and export the data as .csv file. The file can be used in MS Excel for further data analysis.
10. Repeat Step 8-9 with string of different length.
11. Optional: Repeat the experiment with added mass on the pendulum.
12. Switch off the accelerometer by pressing the power button twice.

<u>Data Analysis</u>

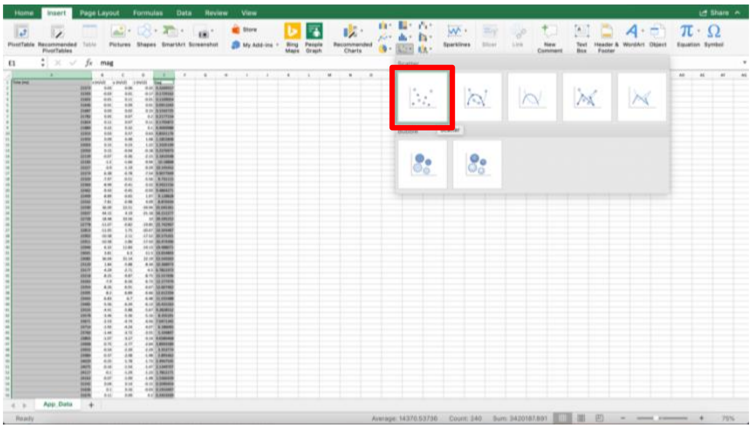
13. Send the .csv files to a computer. Open the .csv file with Excel. (Change the file format to .csv if necessary.) Reverse the time sequence of the recorded data.

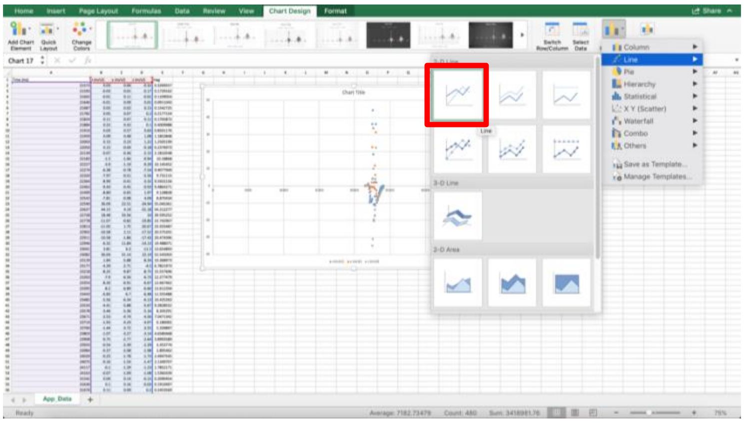


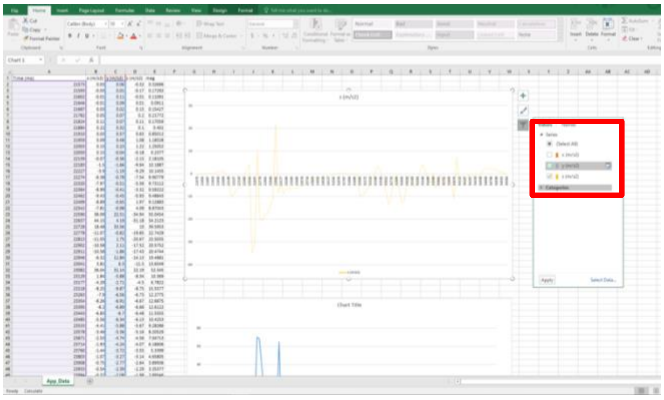


14. Highlight the needed data columns and plot a “scatter” chart with Excel. Connect the data points as line. Then display the desired axis by filtering out the others.
15. From the graphs, analyze the periodic motion. Find the period T.

16. Plot a graph of period T against the square root of length of string $\sqrt{l}$. Determine their relationship.







Data

Period T with different square root of string length $\sqrt{l}$

Length of String *l* (m)	Square Root of l ($m^{0.5}$)	Period T (s)

$\frac{2\pi}{\sqrt{g}}$ = ______________________________________

Slope of graph = ____________________________

Discussion

1. What is the relationship of the period and the square root of string length?
2. Is the proportionality consistent to the expected value of $\frac{2\pi}{\sqrt{g}}$?
3. What will happen if additional mass is added on the pendulum?
4. What would happen if the experiment is identically repeated on Mars, with gravity of around 3.72 ms^{-2}?
5. From the graph of SHM, what happens to the amplitude of the motion along time?
6. What are the possible errors of the experiment? How can we improve to reduce the errors?